TAKE IT FURTHER

The Language of Physics

There is a relationship between rotational motion and circular motion. When a solid object (such as a Ferris wheel) undergoes rotational motion about its fixed axis, a point on the rotating object (such as a light bulb on a Ferris wheel) undergoes circular motion.

TEACH FROM VISUALS

FIGURE 1 Point out that the angle through which the light bulb moves is related to the distance around the circle that the light bulb moves (the arc length).

Ask If the light bulb moves through an angle twice as large as the one shown in (b), how would the new arc length compare with the arc length shown?

Answer: The new arc length would be twice as large.

TEACH FROM VISUALS

FIGURE 2 Strengthen students' understanding of radian measurement by having them use the values given in Figure 2 to estimate the radian measures not shown. The students can then verify their answers by using the conversion equation shown on the next page.

Ask What is the radian measure equal to 75°?

Answer: $\frac{5}{12}\pi$

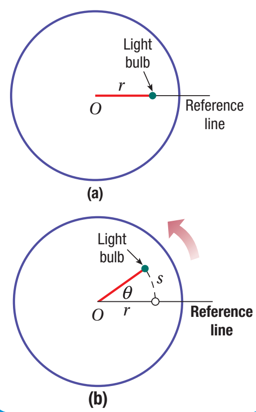
TAKE IT FURTHER

Angular Kinematics

A point on an object that rotates about a fixed axis undergoes circular motion around that axis. The linear quantities introduced previously cannot be used for circular motion because we are considering the rotational motion of an extended object rather than the linear motion of a particle. For this reason, circular motion is described in terms of the change in angular position. All points on a rigid rotating object, except the points on the axis, move through the same angle during any time interval.

FIGURE 1

Circular Motion A light bulb on a rotating Ferris wheel **(a)** begins at a point along a reference line and **(b)** moves through an arc length *s* and therefore through the angle θ.



Measuring Angles with Radians

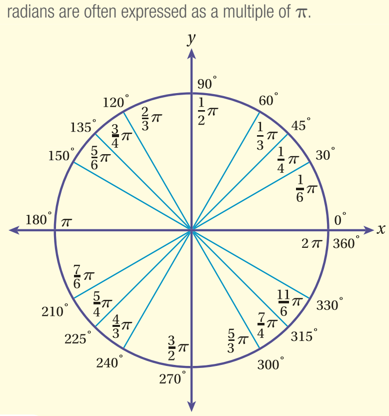
Many of the equations that describe circular motion require that angles be measured in **radians** (rad) rather than in degrees. To see how radians are measured, consider **Figure 1**, which illustrates a light bulb on a rotating Ferris wheel. At $t = 0$, the bulb is on a fixed reference line, as shown in **Figure 1(a).** After a time interval Δt, the bulb advances to a new position, as shown in **Figure 1(b).** In this time interval, the line from the center to the bulb (depicted with a red line in both diagrams) moved through the angle θ with respect to the reference line. Likewise, the bulb moved a distance s, measured along the circumference of the circle; s is the *arc length*.

In general, any angle θ measured in radians is defined by the following equation:

$$\theta = \frac{\text{arc length}}{\text{radius}} = \frac{s}{r}$$

FIGURE 2

Angular Motion Angular motion is measured in units of radians. Because there are 2π radians in a full circle, radians are often expressed as a multiple of π.



Note that if the arc length, s, is equal to the length of the radius, r, the angle θ swept by r is equal to 1 rad. Because θ is the ratio of an arc length (a distance) to the length of the radius (also a distance), the units cancel and the abbreviation *rad* is substituted in their place. In other words, the radian is a pure number, with no dimensions.

When the bulb on the Ferris wheel moves through an angle of 360° (one revolution of the wheel), the arc length s is equal to the circumference of the circle, or $2\pi r$. Substituting this value for s into the equation above gives the corresponding angle in radians.

$$\theta = \frac{s}{r} = \frac{2\pi r}{r} = 2\pi \text{ rad}$$

Thus, 360° equals 2π rad, or one complete revolution. In other words, one revolution corresponds to an angle of approximately $2(3.14) = 6.28$ rad. **Figure 2** on the previous page depicts a circle marked with both radians and degrees.

It follows that any angle in degrees can be converted to an angle in radians by multiplying the angle measured in degrees by $2\pi/360°$. In this way, the degrees cancel out and the measurement is left in radians. The conversion relationship can be simplified as follows:

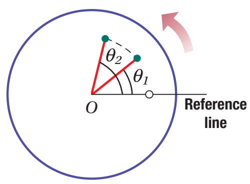
$$\theta\,(\text{rad}) = \frac{\pi}{180°}\,\theta\,(\text{deg})$$

Angular Displacement

Just as an angle in radians is the ratio of the arc length to the radius, the **angular displacement** traveled by the bulb on the Ferris wheel is the change in the arc length, Δs, divided by the distance of the bulb from the axis of rotation. This relationship is depicted in **Figure 3.**

FIGURE 3

Angular Displacement A light bulb on a rotating Ferris wheel rotates through an angular displacement of $\Delta\theta = \theta_2 - \theta_1$.



Angular Displacement

$$\Delta\theta = \frac{\Delta s}{r}$$

$$\textbf{angular displacement (in radians)} = \frac{\textbf{change in arc length}}{\textbf{distance from axis}}$$

This equation is similar to the equation for linear displacement in that this equation denotes a change in position. The difference is that this equation gives a change in *angular* position rather than a change in *linear* position.

For the purposes of this textbook, when a rotating object is viewed from above, the arc length, s, is considered positive when the point rotates counterclockwise and negative when it rotates clockwise. In other words, $\Delta\theta$ is positive when the object rotates counterclockwise and negative when the object rotates clockwise.

QuickLAB — RADIANS AND ARC LENGTH

Use the compass to draw a circle on a sheet of paper, and mark the center point of the circle. Measure the radius of the circle, and cut several pieces of wire equal to the length of this radius. Bend the pieces of wire, and lay them along the circle you drew with your compass. Approximately how many pieces of wire do you use to go all the way around the circle? Draw lines from the center of the circle to each end of one of the wires. Note that the angle between these two lines equals 1 rad.

How many of these angles are there in this circle? Repeat the experiment with a larger circle, and compare the results of each trial.

MATERIALS
- drawing compass
- paper
- thin wire
- wire cutters or scissors

SAFETY

Cut ends of wire are sharp. Cut and handle wire carefully.

Classroom Practice

ANGULAR DISPLACEMENT

Earth has an equatorial radius of approximately 6380 km and rotates 360° every 24 h.

a. What is the angular displacement (in degrees) of a person standing at the equator for 1.0 h?

b. Convert this angular displacement to radians.

c. What is the arc length traveled by this person?

Answers:

a. 15°

b. 0.26 rad

c. approximately 1700 km

QuickLab

TEACHER'S NOTES

Students should find the same results with both circles. In each case, it takes approximately 6 pieces of wire ($6r$) to go around the circle because the circumference $= 2\pi r \approx 6r$.

Homework Options This QuickLab can easily be performed outside of the physics lab room.

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TAKE IT FURTHER

Classroom Practice

ANGULAR VELOCITY

An Indy car can complete 120 laps in 1.5 h. Even though the track is an oval rather than a circle, you can still find the average angular speed. Calculate the average angular speed of the Indy car.

Answer: 0.14 rad/s

ANGULAR ACCELERATION

A top that is spinning at 15 rev/s spins for 55 s before coming to a stop. What is the average angular acceleration of the top while it is slowing?

Answer: -1.7 rad/s^2

Angular Velocity

Angular velocity is defined in a manner similar to that for linear velocity. The average angular velocity of a rotating rigid object is the ratio of the angular displacement, $\Delta\theta$, to the corresponding time interval, Δt. Thus, angular velocity describes how quickly the rotation occurs. Angular velocity is abbreviated as ω_{avg} (ω is the Greek letter *omega*).

Angular Velocity

$$\omega_{avg} = \frac{\Delta\theta}{\Delta t}$$

$$\textbf{average angular velocity} = \frac{\textbf{angular displacement}}{\textbf{time interval}}$$

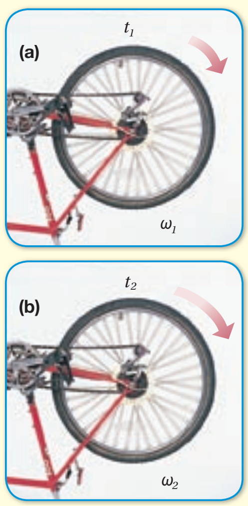
Angular velocity is given in units of radians per second (rad/s). Sometimes, angular velocities are given in revolutions per unit time. Recall that 1 rev = 2π rad. The magnitude of angular velocity is called *angular speed*.

Angular Acceleration

Figure 4 shows a bicycle turned upside down so that a repairperson can work on the rear wheel. The bicycle pedals are turned so that at time t_1 the wheel has angular velocity ω_1, as shown in **Figure 4(a).** At a later time, t_2, it has angular velocity ω_2, as shown in **Figure 4(b).** Because the angular velocity is changing, there is an **angular acceleration.** The average angular acceleration, α_{avg} (α is the Greek letter *alpha*), of an object is given by the relationship shown below. Angular acceleration has the units radians per second per second (rad/s^2).

FIGURE 4

Angular Acceleration
An accelerating bicycle wheel rotates with **(a)** an angular velocity ω_1 at time t_1 and **(b)** an angular velocity ω_2 at time t_2. Thus, the wheel has an angular acceleration.



Angular Acceleration

$$\alpha_{avg} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t}$$

$$\textbf{average angular acceleration} = \frac{\textbf{change in angular velocity}}{\textbf{time interval}}$$

The relationships between the signs of angular displacement, angular velocity, and angular acceleration are similar to those of the related linear quantities. As discussed earlier, by convention, angular displacement is positive when an object rotates counterclockwise and negative when an object rotates clockwise. Thus, by definition, angular velocity is also positive when an object rotates counterclockwise and negative when an object rotates clockwise. Angular acceleration has the same sign as the angular velocity when it increases the magnitude of the angular velocity, and the opposite sign when it decreases the magnitude.

If a point on the rim of a bicycle wheel had an angular velocity greater than a point nearer the center, the shape of the wheel would be changing. Thus, for a rotating object to remain rigid, as does a bicycle wheel or a Ferris wheel, every portion of the object must have the same angular velocity and the same angular acceleration. This fact is precisely what makes angular velocity and angular acceleration so useful for describing rotational motion.

Kinematic Equations for Constant Angular Acceleration

All of the equations for rotational motion defined thus far are analogous to the linear quantities defined in the chapter "Motion in One Dimension." For example, consider the following two equations:

$$\omega_{avg} = \frac{\theta_f - \theta_i}{t_f - t_i} = \frac{\Delta\theta}{\Delta t} \qquad v_{avg} = \frac{x_f - x_i}{t_f - t_i} = \frac{\Delta x}{\Delta t}$$

The equations are similar, with θ replacing x and ω replacing v. The correlations between angular and linear variables are shown in **Figure 5.**

In light of the similarities between variables in linear motion and those in rotational motion, it should be no surprise that the kinematic equations of rotational motion are similar to the linear kinematic equations. The equations of rotational kinematics under constant angular acceleration are summarized in **Figure 6,** along with the corresponding equations for linear motion under constant acceleration. The rotational motion equations apply only for objects rotating about a fixed axis with constant angular acceleration.

FIGURE 5

ANGULAR SUBSTITUTES FOR LINEAR QUANTITIES

Angular	Linear
θ	x
ω	v
α	a

FIGURE 6

ROTATIONAL AND LINEAR KINEMATIC EQUATIONS

Rotational motion with constant angular acceleration	Linear motion with constant acceleration
$\omega_f = \omega_i + \alpha\Delta t$	$v_f = v_i + a\Delta t$
$\Delta\theta = \omega_i\Delta t + \frac{1}{2}\alpha(\Delta t)^2$	$\Delta x = v_i\Delta t + \frac{1}{2}a(\Delta t)^2$
$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$	$v_f^2 = v_i^2 + 2a\Delta x$
$\Delta\theta = \frac{1}{2}(\omega_i + \omega_f)\Delta t$	$\Delta x = \frac{1}{2}(v_i + v_f)\Delta t$

The quantity ω in these equations represents the *instantaneous angular velocity* of the rotating object rather than the average angular velocity.

Demonstration

EQUAL ANGULAR SPEED

Purpose Illustrate that angular speed is constant at any radius for a rigid extended object.

Materials record player/turntable or bicycle, tape, colored markers

Procedure Use the tape and markers to make two brightly-colored flags and attach them to the turntable so that one flag is near the rim and the other is near, but not at, the center. (Alternatively, if a turntable is not available, you can use a bicycle wheel.) Start the turntable at a moderately slow speed ($33\frac{1}{3}$ rpm) so that the flags are easily observed. Have students note the rotational speed of both flags. Point out that each flag makes a complete rotation in the same amount of time. Change speeds on the turntable and repeat the observations.

Classroom Practice

ANGULAR KINEMATICS

A barrel is given a downhill rolling start of 1.5 rad/s at the top of a hill. Assume a constant angular acceleration of 2.9 rad/s^2.

a. If the barrel takes 11.5 s to get to the bottom of the hill, what is the final angular speed of the barrel?

b. What angular displacement does the barrel experience during the 11.5 s ride?

Answers:

a. 35 rad/s

b. 2.1×10^2 rad