Key Terms
momentum
impulse

Linear Momentum

When a soccer player heads a moving ball during a game, the ball’s velocity changes rapidly. After the ball is struck, the ball’s speed and the direction of the ball’s motion change. The ball moves across the soccer field with a different speed than it had and in a different direction than it was traveling before the collision.

The quantities and kinematic equations describing one-dimensional motion predict the motion of the ball before and after the ball is struck. The concept of force and Newton’s laws can be used to calculate how the motion of the ball changes when the ball is struck. In this chapter, we will examine how the force and the duration of the collision between the ball and the soccer player affect the motion of the ball.

Momentum is mass times velocity.

To address such issues, we need a new concept, momentum.

Momentum is a word we use every day in a variety of situations. In physics this word has a specific meaning. The linear momentum of an object of mass $m$ moving with a velocity $v$ is defined as the product of the mass and the velocity. Momentum is represented by the symbol $p$.

$$p = mv$$

As its definition shows, momentum is a vector quantity, with its direction matching that of the velocity. Momentum has dimensions mass $\times$ length/time, and its SI units are kilogram-meters per second $(\text{kg} \cdot \text{m/s})$.

If you think about some examples of the way the word momentum is used in everyday speech, you will see that the physics definition conveys a similar meaning. Imagine coasting down a hill of uniform slope on your bike without pedaling or using the brakes, as shown in Figure 1.1. Because of the force of gravity, you will accelerate; that is, your velocity will increase with time. This idea is often expressed by saying that you are ”picking up speed” or ”gathering momentum.” The faster you move, the more momentum you have and the more difficult it is to come to a stop.
Imagine rolling a bowling ball down one lane at a bowling alley and rolling a playground ball down another lane at the same speed. The more massive bowling ball exerts more force on the pins than the playground ball exerts because the bowling ball has more momentum than the playground ball. When we think of a massive object moving at a high velocity, we often say that the object has a large momentum. A less massive object with the same velocity has a smaller momentum.

On the other hand, a small object moving with a very high velocity may have a larger momentum than a more massive object that is moving slowly does. For example, small hailstones falling from very high clouds can have enough momentum to hurt you or cause serious damage to cars and buildings.

Momentum

Sample Problem A A 2250 kg pickup truck has a velocity of 25 m/s to the east. What is the momentum of the truck?

1. **ANALYZE**
   
   Given:  
   
   
   $m = 2250 \text{ kg}$
   
   $v = 25 \text{ m/s to the east}$
   
   **Unknown:**  
   
   $p = ?$
   
2. **SOLVE**

   Use the definition of momentum.

   $$p = mv = (2250 \text{ kg})(25 \text{ m/s east})$$

   $$p = 5.6 \times 10^4 \text{ kg\cdot m/s to the east}$$

**Practice**

1. A deer with a mass of 146 kg is running head-on toward you with a speed of 17 m/s. You are going north. Find the momentum of the deer.

2. A 21 kg child on a 5.9 kg bike is riding with a velocity of 4.5 m/s to the northwest.
   a. What is the total momentum of the child and the bike together?
   b. What is the momentum of the child?
   c. What is the momentum of the bike?

3. What velocity must a 1210 kg car have in order to have the same momentum as the pickup truck in Sample Problem A?

**Answers**

Practice A

1. $2.5 \times 10^3 \text{ kg\cdot m/s to the south}$

2. a. $1.2 \times 10^2 \text{ kg\cdot m/s to the northwest}$
   b. $94 \text{ kg\cdot m/s to the northwest}$
   c. $27 \text{ kg\cdot m/s to the northwest}$

3. $46 \text{ m/s to the east}$

**PROBLEM GUIDE A**

Use this guide to assign problems.

SE = Student Edition Textbook

PW = Sample Problem Set I (online)

PB = Sample Problem Set II (online)

Solving for:

- $p$
  
  SE Sample, 1–2; Ch. Rvw. 11, 37*
  
  PW 5–6
  
  PB 5–7

- $m$
  
  SE Ch. Rvw. 36*
  
  PW Sample, 1–2
  
  PB 8–10

- $v$
  
  SE Ch. Rvw. 35, 36*
  
  PW 3–4
  
  PB Sample, 1–4

*Challenging Problem

**TAKE IT FURTHER**

Ask students how to find the velocity of an object when they are given its momentum. Present an experiment in which an object with a mass of 32.22 kg experiences a momentum of $82.23 \text{ kg\cdot m/s}$ and ask students to find the velocity of the object. Have them replace the given measures in the equation and solve $82.23 = 32.22v$ for $v$.

$v = \frac{82.23}{32.22} \text{ m/s} = 2.55 \text{ m/s}$
A change in momentum takes force and time.

Figure 1.2 shows a player stopping a moving soccer ball. In a given time interval, he must exert more force to stop a fast ball than to stop a ball that is moving more slowly. Now imagine a toy truck and a real dump truck rolling across a smooth surface with the same velocity. It would take much more force to stop the massive dump truck than to stop the toy truck in the same time interval. You have probably also noticed that a slow-moving ball causes no discomfort when you catch it, while a fast-moving ball stings your hands when you catch it. The fast ball stings because it exerts more force on your hand than the slow-moving ball does.

From examples like these, we see that a change in momentum is closely related to force. In fact, when Newton first expressed his second law mathematically, he wrote it not as \( F = ma \), but in the following form.

\[
F = \frac{\Delta p}{\Delta t}
\]

force = change in momentum  ___   time interval

We can rearrange this equation to find the change in momentum in terms of the net external force and the time interval required to make this change.

**Impulse-Momentum Theorem**

\[
F \Delta t = \Delta p \quad \text{or} \quad F \Delta t = \Delta p = m(v_f - v_i)
\]

force \times time interval = change in momentum

This equation states that a net external force, \( F \), applied to an object for a certain time interval, \( \Delta t \), will cause a change in the object’s momentum equal to the product of the force and the time interval. In simple terms, a small force acting for a long time can produce the same change in momentum as a large force acting for a short time. In this book, all forces exerted on an object are assumed to be constant unless otherwise stated.

The expression \( F \Delta t = \Delta p \) is called the impulse-momentum theorem. The term on the left side of the equation, \( F \Delta t \), is called the impulse of the force \( F \) for the time interval \( \Delta t \).

The equation \( F \Delta t = \Delta p \) explains why proper technique is important in so many sports, from karate and billiards to softball and croquet. For example, when a batter hits a ball, the ball will experience a greater change in momentum if the batter keeps the bat in contact with the ball for a longer time. Extending the time interval over which a constant force is applied allows a smaller force to cause a greater change in momentum than would result if the force were applied for a very short time. You may have noticed this fact when pushing a full shopping cart or moving furniture.

**DEMONSTRATION**

**Purpose** Show that changes in momentum are caused by forces.

**Materials** dynamics cart

**Procedure** Have the students observe the cart at rest. Ask the students the value of the momentum of the cart when it is at rest. Zero. Now push on the cart and ask what has happened to the momentum of the cart. Its momentum has increased. How was the cart’s momentum changed? An external force was applied. Stop the cart as it moves across the table. Again, ask the students how the momentum of the cart was changed. An external force was applied.

**ENGLISH LEARNERS**

Students who are interested in sports may be familiar with the concept of “follow-through.” This is the technique that is being referred to in the last paragraph on this page. Follow-through is what enables batters to keep the bat in contact with the ball for the longest time possible.
**Force and Impulse**

**Sample Problem B** A 1400 kg car moving westward with a velocity of 15 m/s collides with a utility pole and is brought to rest in 0.30 s. Find the force exerted on the car during the collision.

1. **ANALYZE**
   - **Given:**
     - \( m = 1400 \text{ kg} \)
     - \( v_i = 15 \text{ m/s} \) to the west, \( v_f = -15 \text{ m/s} \)
     - \( \Delta t = 0.30 \text{ s} \)
   - **Unknown:** \( F = ? \)

2. **SOLVE**
   - Use the impulse-momentum theorem.
   - \( F\Delta t = \Delta p = m(v_f - v_i) \)
   - \( F = \frac{m(v_f - v_i)}{\Delta t} \)
   - \( F = \frac{(1400 \text{ kg})(0 \text{ m/s}) - (1400 \text{ kg})(-15 \text{ m/s})}{0.30 \text{ s}} = \frac{21000 \text{ kg} \cdot \text{m/s}}{0.30 \text{ s}} = 7.0 \times 10^4 \text{ N} \)
   - **Answer:** \( F = 7.0 \times 10^4 \text{ N to the east} \)

**Practice**

1. A 0.50 kg football is thrown with a velocity of 15 m/s to the right. A stationary receiver catches the ball and brings it to rest in 0.020 s. What is the force exerted on the ball by the receiver?

2. An 82 kg man drops from rest on a diving board 3.0 m above the surface of the water and comes to rest 0.55 s after reaching the water. What is the net force on the diver as he is brought to rest?

3. A 0.40 kg soccer ball approaches a player horizontally with a velocity of 18 m/s to the north. The player strikes the ball and causes it to move in the opposite direction with a velocity of 22 m/s. What impulse was delivered to the ball by the player?

4. A 0.50 kg object is at rest. A 3.00 N force to the right acts on the object during a time interval of 1.50 s.
   - a. What is the velocity of the object at the end of this interval?
   - b. At the end of this interval, a constant force of 4.00 N to the left is applied for 3.00 s. What is the velocity at the end of the 3.00 s?

**Answers**

Practice B

1. \( 3.8 \times 10^3 \text{ N to the left} \)
2. \( 1.1 \times 10^3 \text{ N upward} \)
3. \( 16 \text{ kg} \cdot \text{m/s to the south} \)
4. a. \( 9.0 \text{ m/s to the right} \)
   - b. \( 15 \text{ m/s to the left} \)
Stopping times and distances depend on the impulse-momentum theorem.

Highway safety engineers use the impulse-momentum theorem to determine stopping distances and safe following distances for cars and trucks. For example, the truck hauling a load of bricks in Figure 1.3 has twice the mass of the other truck, which has no load. Therefore, if both are traveling at 48 km/h, the loaded truck has twice as much momentum as the unloaded truck. If we assume that the brakes on each truck exert about the same force, we find that the stopping time is two times longer for the loaded truck than for the unloaded truck, and the stopping distance for the loaded truck is two times greater than the stopping distance for the truck without a load.

**Stopping Distances**
The loaded truck must undergo a greater change in momentum in order to stop than the truck without a load.

**FIGURE 1.3**

**Stopping Distance**

**Sample Problem C**

A 2240 kg car traveling to the west slows down uniformly from 20.0 m/s to 5.00 m/s. How long does it take the car to decelerate if the force on the car is 8410 N to the east? How far does the car travel during the deceleration?

**ANALYZE**

Given:

- \( m = 2240 \text{ kg} \)
- \( v_i = 20.0 \text{ m/s} \) to the west, \( v_f = -20 \text{ m/s} \)
- \( v_i = 5.00 \text{ m/s} \) to the west, \( v_f = -5.00 \text{ m/s} \)
- \( F = 8410 \text{ N} \) to the east, \( F = +8410 \text{ N} \)

Unknown:

- \( \Delta t = ? \)
- \( \Delta x = ? \)

**SOLVE**

Use the impulse-momentum theorem.

\[
F \Delta t = \Delta p
\]

\[
\Delta t = \frac{\Delta p}{F} = \frac{mv_f - mv_i}{F}
\]

\[
\Delta t = \frac{(2240 \text{ kg})(-5.00 \text{ m/s}) - (2240 \text{ kg})(-20.0 \text{ m/s})}{8410 \text{ kg} \cdot \text{m/s}^2}
\]

\[
\Delta t = 4.00 \text{ s}
\]

\[
\Delta x = \frac{1}{2} (v_i + v_f) \Delta t
\]

\[
\Delta x = \frac{1}{2} (-20.0 \text{ m/s} - 5.00 \text{ m/s})(4.00 \text{ s})
\]

\[
\Delta x = -50.0 \text{ m} = 50.0 \text{ m} \text{ to the west}
\]

**TAKE IT FURTHER**

Explain to students that if the velocities of an object before and after acceleration are known, we can find the mass of the object. Present the following scenario and have them find the mass of the object:

A cart is moving with a velocity of 4.62 m/s. In a period of 8.32 s, a force of 12.24 N is applied to the cart. As a result, the cart accelerates, and its velocity reaches 24.21 m/s. What is the mass of the cart? 5.20 kg
Stopping Distance (continued)

**Practice**

1. How long would the car in Sample Problem C take to come to a stop from its initial velocity of 20.0 m/s to the west? How far would the car move before stopping? Assume a constant acceleration.

2. A 2500 kg car traveling to the north is slowed down uniformly from an initial velocity of 20.0 m/s by a 6250 N braking force acting opposite the car’s motion. Use the impulse-momentum theorem to answer the following questions:
   a. What is the car’s velocity after 2.50 s?
   b. How far does the car move during 2.50 s?
   c. How long does it take the car to come to a complete stop?

3. Assume that the car in Sample Problem C has a mass of 3250 kg.
   a. How much force would be required to cause the same acceleration as in item 1? Use the impulse-momentum theorem.
   b. How far would the car move before stopping? (Use the force found in a.)

**Force is reduced when the time interval of an impact is increased.**

The impulse-momentum theorem is used to design safety equipment that reduces the force exerted on the human body during collisions. Examples of this are the nets and giant air mattresses firefighters use to catch people who must jump out of tall burning buildings. The relationship is also used to design sports equipment and games.

Figure 1.4 shows an Inupiat family playing a traditional game. Common sense tells us that it is much better for the girl to fall onto the outstretched blanket than onto the hard ground. In both cases, however, the change in momentum of the falling girl is exactly the same. The difference is that the blanket “gives way” and extends the time of collision so that the change in the girl’s momentum occurs over a longer time interval. A longer time interval requires a smaller force to achieve the same change in the girl’s momentum. Therefore, the force exerted on the girl when she lands on the outstretched blanket is less than the force would be if she were to land on the ground.

**Figure 1.4**

*Increasing the Time of Impact* In this game, the girl is protected from injury because the blanket reduces the force of the collision by allowing it to take place over a longer time interval.

**Answers**

**Practice C**

1. 5.33 s; 53.3 m to the west
2. a. 14 m/s to the north  
   b. 42 m to the north  
   c. 8.0 s
3. a. $1.22 \times 10^4$ N to the east  
   b. 53.3 m to the west
Reviewing Main Ideas

1. The speed of a particle is doubled.
   a. By what factor is its momentum changed?
   b. What happens to its kinetic energy?

2. A pitcher claims he can throw a 0.145 kg baseball with as much momentum as a speeding bullet. Assume that a 3.00 g bullet moves at a speed of \(1.50 \times 10^3\) m/s.
   a. What must the baseball’s speed be if the pitcher’s claim is valid?
   b. Which has greater kinetic energy, the ball or the bullet?

3. A 0.42 kg soccer ball is moving downfield with a velocity of 12 m/s. A player kicks the ball so that it has a final velocity of 18 m/s downfield.
   a. What is the change in the ball’s momentum?
   b. Find the constant force exerted by the player’s foot on the ball if the two are in contact for 0.020 s.

Critical Thinking

4. When a force is exerted on an object, does a large force always produce a larger change in the object’s momentum than a smaller force does? Explain.

5. What is the relationship between impulse and momentum?