Conservation of Energy

Key Term
mechanical energy

Conserved Quantities
When we say that something is conserved, we mean that it remains constant. If we have a certain amount of a conserved quantity at some instant of time, we will have the same amount of that quantity at a later time. This does not mean that the quantity cannot change form during that time, but if we consider all the forms that the quantity can take, we will find that we always have the same amount.

For example, the amount of money you now have is not a conserved quantity because it is likely to change over time. For the moment, however, let us assume that you do not spend the money you have, so your money is conserved. This means that if you have a dollar in your pocket, you will always have that same amount, although it may change form. One day it may be in the form of a bill. The next day you may have a hundred pennies, and the next day you may have an assortment of dimes and nickels. But when you total the change, you always have the equivalent of a dollar. It would be nice if money were like this, but of course it isn't. Because your money is often acquired and spent, it is not a conserved quantity.

An example of a conserved quantity that you are already familiar with is mass. For instance, imagine that a light bulb is dropped on the floor and shatters into many pieces. No matter how the bulb shatters, the total mass of all of the pieces together is the same as the mass of the intact light bulb because mass is conserved.

Mechanical Energy
We have seen examples of objects that have either kinetic or potential energy. The description of the motion of many objects, however, often involves a combination of kinetic and potential energy as well as different forms of potential energy. Situations involving a combination of these different forms of energy can often be analyzed simply. For example, consider the motion of the different parts of a pendulum clock. The pendulum swings back and forth. At the highest point of its swing, there is only gravitational potential energy associated with its position. At other points in its swing, the pendulum is in motion, so it has kinetic energy as well. Elastic potential energy is also present in the many springs that are part of the inner workings of the clock. The motion of the pendulum in a clock is shown in Figure 3.1.

Differentiated Instruction

ENGLISH LEARNERS
The term conservation may be familiar to students in the context of environmental concerns, related to the concepts of protecting and preserving. Use this knowledge to discuss conservation of energy. When energy is changed from one form to another, the total amount of energy is preserved; it remains unchanged. Ask how the concept of conserved quantities in physics is related to the environmental concept of conservation.
Energy can be classified in a number of ways.

Analyzing situations involving kinetic, gravitational potential, and elastic potential energy is relatively simple. Unfortunately, analyzing situations involving other forms of energy—such as chemical potential energy—is not as easy.

We can ignore these other forms of energy if their influence is negligible or if they are not relevant to the situation being analyzed. In most situations that we are concerned with, these forms of energy are not involved in the motion of objects. In ignoring these other forms of energy, we will find it useful to define a quantity called mechanical energy.

**Mechanical energy** is the sum of kinetic energy and all forms of potential energy associated with an object or group of objects.

\[ ME = KE + \Sigma PE \]

All energy, such as nuclear, chemical, internal, and electrical, that is not mechanical energy is classified as nonmechanical energy. Do not be confused by the term mechanical energy. It is not a unique form of energy. It is merely a way of classifying energy, as shown in Figure 3.2. As you learn about new forms of energy in this book, you will be able to add them to this chart.

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**Mechanical energy is often conserved.**

Imagine a 75 g egg located on a countertop 1.0 m above the ground, as shown in Figure 3.3. The egg is knocked off the edge and falls to the ground. Because the acceleration of the egg is constant as it falls, you can use the kinematic formulas to determine the speed of the egg and the distance the egg has fallen at any subsequent time. The distance fallen can then be subtracted from the initial height to find the height of the egg above the ground at any subsequent time. For example, after 0.10 s, the egg has a speed of 0.98 m/s and has fallen a distance of 0.05 m, corresponding to a height above the ground of 0.95 m. Once the egg’s speed and its height above the ground are known as a function of time, you can use what you have learned in this chapter to calculate both the kinetic energy of the egg and the gravitational potential energy associated with the position of the egg at any subsequent time. Adding the kinetic and potential energy gives the total mechanical energy at each position.

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**Differentiated Instruction**

**Below Level**

Students may have difficulty understanding the energy classifications in Figure 3.2. Put the examples below on the board and help students identify which chain of energy classifications are correct and which are incorrect.

- Mechanical
- Gravitational
- Elastic
- Internal
- Elastic
- Mechanical
- Mechanical
- Nuclear
- Kinetic
- Mechanical
- Potential
- Gravitational
In the absence of friction, the total mechanical energy remains the same. This principle is called conservation of mechanical energy. Although the amount of mechanical energy is constant, mechanical energy itself can change form. For instance, consider the forms of energy for the falling egg, as shown in Figure 3.4. As the egg falls, the potential energy is continuously converted into kinetic energy. If the egg were thrown up in the air, kinetic energy would be converted into gravitational potential energy. In either case, mechanical energy is conserved. The conservation of mechanical energy can be written symbolically as follows:

\[
ME_i = ME_f
\]

initial mechanical energy = final mechanical energy
(in the absence of friction)

### Conservation of Mechanical Energy

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>Height (m)</th>
<th>Speed (m/s)</th>
<th>(PE_g) (J)</th>
<th>KE (J)</th>
<th>ME (J)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>1.0</td>
<td>0.00</td>
<td>0.74</td>
<td>0.00</td>
<td>0.74</td>
</tr>
<tr>
<td>0.10</td>
<td>0.95</td>
<td>0.98</td>
<td>0.70</td>
<td>0.036</td>
<td>0.74</td>
</tr>
<tr>
<td>0.20</td>
<td>0.80</td>
<td>2.0</td>
<td>0.59</td>
<td>0.15</td>
<td>0.74</td>
</tr>
<tr>
<td>0.30</td>
<td>0.56</td>
<td>2.9</td>
<td>0.41</td>
<td>0.33</td>
<td>0.74</td>
</tr>
<tr>
<td>0.40</td>
<td>0.22</td>
<td>3.9</td>
<td>0.16</td>
<td>0.58</td>
<td>0.74</td>
</tr>
</tbody>
</table>

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### QuickLab

#### MATERIALS
- medium-sized spring (spring balance)
- assortment of small balls, each having a different mass
- ruler
- tape
- scale or balance

#### SAFETY
Students should wear goggles to perform this lab.

#### TEACHER'S NOTES
This activity is meant to demonstrate energy transfer (from the spring to the ball) and the conservation of mechanical energy.

The lab is most effective when the balls have significantly different masses and when the spring is compressed the same amount in each case.

Because the system for all cases has the same \(ME_i = \frac{1}{2}kx^2\)—which is converted into \(ME_f = mgh_f\)—balls with a larger mass will achieve a lower height.

Point out that if the measurements are reliable, they can be used to determine the spring constant.

#### PRE-AP
Manipulate the last formula as shown below to develop a relationship between initial velocity, final velocity, initial height, and final height.

Show how the conservation law is independent of mass:

\[
\frac{1}{2}mv^2_i + mgh_i = \frac{1}{2}mv^2_f + mgh_f
\]

\[
m\left(\frac{1}{2}v^2_i + gh_i\right) = m\left(\frac{1}{2}v^2_f + gh_f\right)
\]

\[
\frac{1}{2}v^2_i + gh_i = \frac{1}{2}v^2_f + gh_f
\]
The mathematical expression for the conservation of mechanical energy depends on the forms of potential energy in a given problem. For instance, if the only force acting on an object is the force of gravity, as in the egg example, the conservation law can be written as follows:

$$\frac{1}{2}mv_i^2 + mgh_i = \frac{1}{2}mv_f^2 + mgh_f$$

If other forces (except friction) are present, simply add the appropriate potential energy terms associated with each force. For instance, if the egg happened to compress or stretch a spring as it fell, the conservation law would also include an elastic potential energy term on each side of the equation.

In situations in which frictional forces are present, the principle of mechanical energy conservation no longer holds because kinetic energy is not simply converted to a form of potential energy. This special situation will be discussed more thoroughly later in this section.

Conservation of Mechanical Energy

Sample Problem E Starting from rest, a child zooms down a frictionless slide from an initial height of 3.00 m. What is her speed at the bottom of the slide? Assume she has a mass of 25.0 kg.

1. **ANALYZE**
   - Given: $h = h_i = 3.00$ m, $m = 25.0$ kg, $v_i = 0.0$ m/s, $h_f = 0$ m
   - Unknown: $v_f = ?$

2. **PLAN**

   Choose an equation or situation:
   - The slide is frictionless, so mechanical energy is conserved. Kinetic energy and gravitational potential energy are the only forms of energy present.
   - $KE = \frac{1}{2}mv^2$, $PE = mgh$

   The zero level chosen for gravitational potential energy is the bottom of the slide. Because the child ends at the zero level, the final gravitational potential energy is zero.

   $PE_{g,f} = 0$
   - The initial gravitational potential energy at the top of the slide is $PE_{g,i} = mgh_i = mgh$
   - Because the child starts at rest, the initial kinetic energy at the top is zero.

   $KE_i = 0$
   - Therefore, the final kinetic energy is as follows:

   $$KE_f = \frac{1}{2}mv_f^2$$

Answers

- a. 0.36 J
- b. 0.36 J
- c. 8.5 m/s
- d. 3.7 m
Conservation of Mechanical Energy (continued)

SOLVE

Substitute the values into the equation:

\[ PE_i = (25.0 \text{ kg})(9.81 \text{ m/s}^2)(3.00 \text{ m}) = 736 \text{ J} \]

\[ KE_f = \frac{1}{2} (25.0 \text{ kg}) v_f^2 \]

Now use the calculated quantities to evaluate the final velocity.

\[ ME_i = ME_f \]
\[ PE_i + KE_i = PE_f + KE_f \]

\[ 736 \text{ J} + 0 \text{ J} = 0 \text{ J} + (0.500)(25.0 \text{ kg}) v_f^2 \]

\[ v_f = 7.67 \text{ m/s} \]

CHECK YOUR WORK

The expression for the square of the final speed can be written as follows:

\[ v_f^2 = \frac{2mgh}{m} = 2gh \]

Notice that the masses cancel, so the final speed does not depend on the mass of the child. This result makes sense because the acceleration of an object due to gravity does not depend on the mass of the object.

Practice

1. A bird is flying with a speed of 18.0 m/s over water when it accidentally drops a 2.00 kg fish. If the altitude of the bird is 5.40 m and friction is disregarded, what is the speed of the fish when it hits the water?

2. A 755 N diver drops from a board 10.0 m above the water’s surface. Find the diver’s speed 5.00 m above the water’s surface. Then find the diver’s speed just before striking the water.

3. If the diver in item 2 leaves the board with an initial upward speed of 2.00 m/s, find the diver’s speed when striking the water.

4. An Olympic runner leaps over a hurdle. If the runner’s initial vertical speed is 2.2 m/s, how much will the runner’s center of mass be raised during the jump?

5. A pendulum bob is released from some initial height such that the speed of the bob at the bottom of the swing is 1.9 m/s. What is the initial height of the bob?

Energy conservation occurs even when acceleration varies.

If the slope of the slide in Sample Problem E was constant, the acceleration along the slide would also be constant and the one-dimensional kinematic formulas could have been used to solve the problem. However, you do not know the shape of the slide. Thus, the acceleration may not be constant, and the kinematic formulas could not be used.

ALTERNATIVE APPROACHES

The process shown in Sample Problem E can be reversed. Rather than calculating each type of energy separately, begin with the conservation of mechanical energy:

\[ ME_i = ME_f \]

Next determine what types of energy are involved and substitute the formulas for each type of energy into the equation.

In this case,

\[ PE_i = KE_f \]
\[ mgh_i = \frac{1}{2} mv_f^2 \]

Solve for \( v_f \) in terms of the other variables, and then substitute the given values into this equation.

\[ v_f^2 = 2gh_i \]
\[ v_f = \sqrt{2gh_i} \]

\[ v_f = \sqrt{2(9.81 \text{ m/s}^2)(3.00 \text{ m})} \]
\[ v_f = 7.67 \text{ m/s} \]
Reviewing Main Ideas

1. If the spring of a jack-in-the-box is compressed a distance of 8.00 cm from its relaxed length and then released, what is the speed of the toy head when the spring returns to its natural length? Assume the mass of the toy head is 50.0 g, the spring constant is 80.0 N/m, and the toy head moves only in the vertical direction. Also disregard the mass of the spring. (Hint: Remember that there are two forms of potential energy in the problem.)

2. You are designing a roller coaster in which a car will be pulled to the top of a hill of height \( h \) and then, starting from a momentary rest, will be released to roll freely down the hill and toward the peak of the next hill, which is 1.1 times as high. Will your design be successful? Explain your answer.

3. Is conservation of mechanical energy likely to hold in these situations?
   a. a hockey puck sliding on a frictionless surface of ice
   b. a toy car rolling on a carpeted floor
   c. a baseball being thrown into the air

Critical Thinking

4. What parts of the kinetic sculpture on the opening pages of this chapter involve the conversion of one form of energy to another? Is mechanical energy conserved in these processes?

But now we can apply a new method to solve such a problem. Because the slide is frictionless, mechanical energy is conserved. We simply equate the initial mechanical energy to the final mechanical energy and ignore all the details in the middle. The shape of the slide is not a contributing factor to the system’s mechanical energy as long as friction can be ignored.

Mechanical energy is not conserved in the presence of friction.

If you have ever used a sanding block to sand a rough surface, such as in Figure 3.5, you may have noticed that you had to keep applying a force to keep the block moving. The reason is that kinetic friction between the moving block and the surface causes the kinetic energy of the block to be converted into a nonmechanical form of energy. As you continue to exert a force on the block, you are replacing the kinetic energy that is lost because of kinetic friction. The observable result of this energy dissipation is that the sanding block and the tabletop become warmer.

In the presence of kinetic friction, nonmechanical energy is no longer negligible and mechanical energy is no longer conserved. This does not mean that energy in general is not conserved—total energy is always conserved. However, the mechanical energy is converted into forms of energy that are much more difficult to account for, and the mechanical energy is therefore considered to be “lost.”

Misconception Alert!

Some students may confuse the conservation of mechanical energy with the general energy conservation law. Point out that although mechanical energy is not always conserved, the total energy is always conserved. For example, as the sanding block’s kinetic energy decreases, energy is transferred to the rough surface in the form of internal energy (this topic will be discussed in the chapter on heat and temperature). As a result, the temperatures of the block and surface increase slightly. The total energy in the system remains constant, although the mechanical energy decreases.

Assess and Reteach

**Assess** Use the Formative Assessment on this page to evaluate student mastery of the section.

**Reteach** For students who need additional instruction, download the Section Study Guide.

**Response to Intervention** To reassess students’ mastery, use the Section Quiz, available to print or to take directly online at HMDScience.com.

Answers to Section Assessment

1. 2.93 m/s
2. No, the roller coaster will not reach the top of the second hill. If the total mechanical energy is constant, the roller coaster will reach its initial height and then begin rolling back down the hill.
3. a. yes
   b. no
   c. yes, if air resistance is disregarded
4. Answers may vary. The downward-sloping track converts potential energy to kinetic energy. Levers employ kinetic energy to increase potential energy. Springs and elastic membranes convert kinetic energy to elastic potential energy and back again. Mechanical energy is not conserved; some energy is lost because of kinetic friction.