Kinetic Energy

Kinetic energy is energy associated with an object in motion. Figure 2.1 shows a cart of mass \( m \) moving to the right on a frictionless air track under the action of a constant net force, \( F \), acting to the right. Because the force is constant, we know from Newton’s second law that the cart moves with a constant acceleration, \( a \). While the force is applied, the cart accelerates from an initial velocity \( v_i \) to a final velocity \( v_f \). If the cart is displaced a distance of \( \Delta x \), the work done by \( F \) during this displacement is

\[
W_{\text{net}} = F \Delta x = ma \Delta x
\]

When you studied one-dimensional motion, you learned that the following relationship holds when an object undergoes constant acceleration:

\[
v_f^2 = v_i^2 + 2a \Delta x
\]

\[
a \Delta x = \frac{v_f^2 - v_i^2}{2}
\]

Substituting this result into the equation \( W_{\text{net}} = ma \Delta x \) gives

\[
W_{\text{net}} = m \left( \frac{v_f^2 - v_i^2}{2} \right)
\]

\[
W_{\text{net}} = \frac{1}{2} mv_f^2 - \frac{1}{2} mv_i^2
\]

The quantity \( \frac{1}{2} mv^2 \) has a special name in physics: kinetic energy. The kinetic energy of an object with mass \( m \) and speed \( v \), when treated as a particle, is given by the expression shown on the next page.
Kinetic Energy

\[ KE = \frac{1}{2} \, mv^2 \]

kinetic energy \( = \frac{1}{2} \times \text{mass} \times (\text{speed})^2 \)

Kinetic energy is a scalar quantity, and the SI unit for kinetic energy (and all other forms of energy) is the joule. Recall that a joule is also used as the basic unit for work.

Kinetic energy depends on both an object’s speed and its mass. If a bowling ball and a volleyball are traveling at the same speed, which do you think has more kinetic energy? You may think that because they are moving with identical speeds they have exactly the same kinetic energy. However, the bowling ball has more kinetic energy than the volleyball traveling at the same speed because the bowling ball has more mass than the volleyball.

Sample Problem B

A 7.00 kg bowling ball moves at 3.00 m/s. How fast must a 2.45 g table-tennis ball move in order to have the same kinetic energy as the bowling ball? Is this speed reasonable for a table-tennis ball in play?

**ANALYZE**

Given:

The subscripts \( b \) and \( t \) indicate the bowling ball and the table-tennis ball, respectively.

\[ m_b = 7.00 \, \text{kg} \quad m_t = 2.45 \, \text{g} \quad v_b = 3.00 \, \text{m/s} \]

Unknown: \( v_t = ? \)

**PLAN**

First, calculate the kinetic energy of the bowling ball.

\[ KE_b = \frac{1}{2} \, m_b \, v_b^2 = \frac{1}{2} \times (7.00 \, \text{kg})(3.00 \, \text{m/s})^2 = 31.5 \, \text{J} \]

Then, solve for the speed of the table-tennis ball having the same kinetic energy as the bowling ball.

\[ KE_t = \frac{1}{2} \, m_t \, v_t^2 = KE_b = 31.5 \, \text{J} \]

**SOLVE**

\[ v_t = \sqrt{\frac{KE_b}{m_t}} = \sqrt{\frac{(2)(31.5 \, \text{J})}{2.45 \times 10^{-3} \, \text{kg}}} \]

\[ v_t = 1.60 \times 10^2 \, \text{m/s} \]

This speed would be very fast for a table-tennis ball.

Problem Solving

DECONSTRUCTING PROBLEMS

Point out to students that in the equation for kinetic energy, kinetic energy is expressed in joules. Be sure students know that in order for the units of energy to work out properly, mass must be in kg and speed must be in m/s. Ask them to calculate the kinetic energy of an object that has a mass of 100 grams and moves at 100 meters per minute. 0.14 J
Kinetic Energy (continued)

Practice

1. Calculate the speed of an 8.0 \times 10^4 \text{ kg} airliner with a kinetic energy of 1.1 \times 10^9 \text{ J}.
2. What is the speed of a 0.145 \text{ kg} baseball if its kinetic energy is 109 \text{ J}?
3. Two bullets have masses of 3.0 \text{ g} and 6.0 \text{ g}, respectively. Both are fired with a speed of 40.0 \text{ m/s}. Which bullet has more kinetic energy? What is the ratio of their kinetic energies?
4. Two 3.0 \text{ g} bullets are fired with speeds of 40.0 \text{ m/s} and 80.0 \text{ m/s}, respectively. What are their kinetic energies? Which bullet has more kinetic energy? What is the ratio of their kinetic energies?
5. A car has a kinetic energy of 4.32 \times 10^5 \text{ J} when traveling at a speed of 23 \text{ m/s}. What is its mass?

The Language of Physics

The symbol \( \Delta \) (the Greek letter delta) is used to denote change. Students should be familiar with this symbol from earlier chapters. Point out that although the context is different, the symbol means the same thing; namely, a difference between two quantities. The subscripts \( i \) and \( f \) used with \( KE \) stand for the initial and final amounts, respectively, of mechanical energy. Thus, \( \Delta KE \) is the difference between \( KE_f \) and \( KE_i \), or \( \Delta KE = KE_f - KE_i \).

The net work done on a body equals its change in kinetic energy. The equation \( W_{net} = \frac{1}{2} mv_f^2 - \frac{1}{2} mv_i^2 \) derived at the beginning of this section says that the net work done on a net force acting on an object is equal to the change in the object’s kinetic energy. This important relationship, known as the work–kinetic energy theorem, is often written as follows:

\[ W_{net} = \Delta KE \]

When you use this theorem, you must include all the forces that do work on the object in calculating the net work done. From this theorem, we see that the speed of the object increases if the net work done is positive, because the final kinetic energy is greater than the initial kinetic energy. The object’s speed decreases if the net work is negative, because the final kinetic energy is less than the initial kinetic energy.

The work–kinetic energy theorem allows us to think of kinetic energy as the work that an object can do while the object changes speed or as the amount of energy stored in the motion of an object. For example, the moving hammer in the ring-the-bell game in Figure 2.2 has kinetic energy and can therefore do work on the puck. The puck can do work against gravity by moving up and striking the bell. When the bell is struck, part of the energy is converted into sound.

Differentiated Instruction

Inclusion

In the ring-the-bell game, part of the energy of the moving puck is converted into sound. The greater the force with which the puck hits the bell, the louder the sound of the bell. Students with hearing impairments may grasp the concept of the relationship between energy and sound. People with profound hearing loss can often feel the vibrations caused by sounds, with louder sounds causing stronger vibrations.
Sample Problem C On a frozen pond, a person kicks a 10.0 kg sled, giving it an initial speed of 2.2 \text{ m/s}. How far does the sled move if the coefficient of kinetic friction between the sled and the ice is 0.10?

**Analyse**

Given:

\[ m = 10.0 \text{ kg} \quad v_i = 2.2 \text{ m/s} \quad v_f = 0 \text{ m/s} \quad \mu_k = 0.10 \]

Unknown:

\[ d = ? \]

Diagram:

![Diagram](image)

**Plan**

Choose an equation or situation:

This problem can be solved using the definition of work and the work-kinetic energy theorem.

\[ W_{net} = F_{net} \cdot d \cos \theta \]

The net work done on the sled is provided by the force of kinetic friction.

\[ W_{net} = F_k \cdot d \cos \theta = \mu_k mgd \cos \theta \]

The force of kinetic friction is in the direction opposite \( d \), so \( \theta = 180^\circ \). Because the sled comes to rest, the final kinetic energy is zero.

\[ W_{net} = \Delta KE = KE_f - KE_i = -\frac{1}{2}mv_i^2 \]

Use the work-kinetic energy theorem, and solve for \( d \).

\[ \frac{-\frac{1}{2}mv_i^2}{\mu_k mg \cos \theta} = d \]

\[ d = \frac{-v_i^2}{2 \mu_k g \cos \theta} \]

**Solve**

Substitute values into the equation:

\[ d = \frac{- (2.2 \text{ m/s})^2}{2(0.10)(9.81 \text{ m/s}^2)(\cos 180^\circ)} \]

\[ d = 2.5 \text{ m} \]

**Check Your Work**

According to Newton’s second law, the acceleration of the sled is about \(-1 \text{ m/s}^2\) and the time it takes the sled to stop is about 2 s. Thus, the distance the sled traveled in the given amount of time should be less than the distance it would have traveled in the absence of friction.

\[ 2.5 \text{ m} < (2.2 \text{ m/s})(2 \text{ s}) = 4.4 \text{ m} \]

---

**Deconstructing Problems**

Simplify the common factors in each fraction:

\[ \frac{-v_i^2}{2 \mu_k g \cos \theta} = d \]

Divide each side of \(-\frac{1}{2}mv_i^2 = \mu_k mgd \cos \theta\) by \( \mu_k mg \cos \theta\):

\[ \frac{2\left(-\frac{1}{2}mv_i^2\right)}{2 \left(\mu_k mg \cos \theta\right)} = \frac{\mu_k mg \cos \theta}{\mu_k mg \cos \theta} \]
The food that you eat provides your body with energy. Your body needs this energy to move your muscles, to maintain a steady internal temperature, and to carry out many other bodily processes. The energy in food is stored as a kind of potential energy in the chemical bonds within sugars and other organic molecules.

When you digest food, some of this energy is released. The energy is then stored again in sugar molecules, usually as glucose. When cells in your body need energy to carry out cellular processes, the cells break down the glucose molecules through a process called cellular respiration. The primary product of cellular respiration is a high-energy molecule called adenosine triphosphate (ATP), which has a significant role in many chemical reactions in cells.

Nutritionists and food scientists use units of Calories to quantify the energy in food. A standard calorie (cal) is defined as the amount of energy required to increase the temperature of 1 mL of water by 1°C, which equals 4.186 joules (J). A food Calorie is actually 1 kilocalorie, or 4186 J.

People who are trying to lose weight often monitor the number of Calories that they eat each day. These people count Calories because the body stores unused energy as fat. Most food labels show the number of Calories in each serving of food. The amount of energy that your body needs each day depends on many factors, including your age, your weight, and the amount of exercise that you get. A typically healthy and active person requires about 1500 to 2000 Calories per day.

### The Energy in Food

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### Problem Solving

#### **TAKE IT FURTHER**

In planning a diet, it is important for students to know that fats, carbohydrates, and proteins contain different numbers of Calories per gram. Fats and oils contain 9 Cal/g, and carbohydrates and proteins contain 4 Cal/g.

Give students this problem: In one day, a person ate 80 grams of fat, 300 grams of carbohydrates, and 56 grams of protein. In order to maintain his weight, this person requires 1900 Calories per day.

Was he above or below his desired Calorie intake for that day? **1964 Cal total; above** For a person who needs to lose weight, does it make more sense to reduce protein intake or fat intake? **Reducing fat reduces more Calories per gram.**
Potential Energy

Consider the balanced boulder shown in Figure 2.3. As long as the boulder remains balanced, it has no kinetic energy. If it becomes unbalanced, it will fall vertically to the desert floor and will gain kinetic energy as it falls. What is the origin of this kinetic energy?

Potential energy is stored energy.

Potential energy is associated with an object that has the potential to move because of its position relative to some other location. Unlike kinetic energy, potential energy depends not only on the properties of an object but also on the object’s interaction with its environment.

Gravitational potential energy depends on height from a zero level.

You learned earlier how gravitational forces influence the motion of a projectile. If an object is thrown up in the air, the force of gravity will eventually cause the object to fall back down. Similarly, the force of gravity will cause the unbalanced boulder in the previous example to fall. The energy associated with an object due to its position relative to a gravitational source is called gravitational potential energy.

Imagine an egg falling off a table. As it falls, it gains kinetic energy. But where does the egg’s kinetic energy come from? It comes from the gravitational potential energy that is associated with the egg’s initial position on the table relative to the floor. Gravitational potential energy can be determined using the following equation:

\[ \text{Gravitational Potential Energy} \]

\[ PE_g = mgh \]

where \( m \) is the mass of the object, \( g \) is the free-fall acceleration, and \( h \) is the height.

The SI unit for gravitational potential energy, like for kinetic energy, is the joule. Note that the definition for gravitational potential energy given here is valid only when the free-fall acceleration is constant over the entire height, such as at any point near Earth’s surface. Furthermore, gravitational potential energy depends on both the height and the free-fall acceleration, neither of which is a property of an object.

Also note that the height, \( h \), is measured from an arbitrary zero level. In the example of the egg, if the floor is the zero level, then \( h \) is the height of the table, and \( mgh \) is the gravitational potential energy relative to the floor. Alternatively, if the table is the zero level, then \( h \) is zero. Thus, the potential energy associated with the egg relative to the table is zero.

Suppose you drop a volleyball from a second-floor roof and it lands on the first-floor roof of an adjacent building (see Figure 2.4). If the height is measured from the ground, the gravitational potential energy is not zero because the ball is still above the ground. But if the height is measured from the first-floor roof, the potential energy is zero when the ball lands on the roof.

\[ \text{Defining Potential Energy with Respect to Position} \]

If \( B \) is the zero level, then all the gravitational potential energy is converted to kinetic energy as the ball falls from \( A \) to \( B \). If \( C \) is the zero level, then only part of the total gravitational potential energy is converted to kinetic energy during the fall from \( A \) to \( B \).

The Language of Physics

In the symbol \( PE_g \), \( PE \) stands for potential energy, and the subscript \( g \) specifies that the source of this potential energy is gravity. Some texts use \( U \) rather than \( PE \) to represent potential energy.

Demonstration

**POTENTIAL ENERGY**

**Purpose** Show that potential energy is stored energy.

**Materials** a racquetball cut in half (note that you may have to trim the cut ball slightly more for this demonstration to work properly)

**Caution** Do not face the area where you drop the ball, because it may rise up high enough to hit you.

**Procedure** Pop the hollow hemisphere of the ball inside out and hold it hollow-side up. Before dropping it from a low height, ask students to predict whether it will bounce back and, if so, approximately how high.

Release the ball. The half ball will pop out on impact with the surface and will bounce up to a greater height with its hollow side facing down. Ask students where the additional gravitational potential energy came from. Elastic potential energy was stored in the half ball when it was inverted inside out.
Gravitational potential energy is associated with an object’s position, so it must be measured relative to some zero level. The zero level is the vertical coordinate at which gravitational potential energy is defined to be zero. This zero level is arbitrary, and it is chosen to make a specific problem easier to solve. In many cases, the statement of the problem suggests what to use as a zero level.

Elastic potential energy depends on distance compressed or stretched. Imagine you are playing with a spring on a tabletop. You push a block into the spring, compressing the spring, and then release the block. The block slides across the tabletop. The kinetic energy of the block came from the stored energy in the compressed spring. This potential energy is called elastic potential energy.

Elastic potential energy is stored in any compressed or stretched object, such as a spring or the stretched strings of a tennis racket or guitar.

The length of a spring when no external forces are acting on it is called the relaxed length of the spring. When an external force compresses or stretches the spring, elastic potential energy is stored in the spring. The amount of energy depends on the distance the spring is compressed or stretched from its relaxed length, as shown in Figure 2.5. Elastic potential energy can be determined using the following equation:

\[ \text{Elastic Potential Energy} \]

\[ PE_{\text{elastic}} = \frac{1}{2} k x^2 \]

The symbol \( k \) is called the spring constant, or force constant. For a flexible spring, the spring constant is small, whereas for a stiff spring, the spring constant is large. Spring constants have units of newtons divided by meters (N/m).

---

**Differentiated Instruction**

**BELOW LEVEL**

Some students do not realize that the potential energy of an object is relative. Point out that the zero level for measuring height is arbitrarily defined in each problem. The potential energy is calculated relative to that level. Ask students how they would calculate the potential energy of a book on their desk relative to the desk, to the classroom floor, and to the roof. The book on the desk has no potential energy relative to the desk. Relative to the classroom floor, the height of the desk would be used to calculate potential energy. Relative to the roof, the gravitational potential energy is negative.
Sample Problem D
A 70.0 kg stuntman is attached to a bungee cord with an unstretched length of 15.0 m. He jumps off a bridge spanning a river from a height of 50.0 m. When he finally stops, the cord has a stretched length of 44.0 m. Treat the stuntman as a point mass, and disregard the weight of the bungee cord. Assuming the spring constant of the bungee cord is 71.8 N/m, what is the total potential energy relative to the water when the man stops falling?

**ANALYZE**

Given:
- \( m = 70.0 \text{ N} \)
- \( k = 71.8 \text{ N/m} \)
- \( g = 9.81 \text{ m/s}^2 \)
- \( h = 50.0 \text{ m} - 44.0 \text{ m} = 6.0 \text{ m} \)
- \( x = 44.0 \text{ m} - 15.0 \text{ m} = 29.0 \text{ m} \)
- \( PE = 0 \text{ J} \) at river level

Unknown: \( PE_{\text{tot}} = ? \)

**Diagram:**

Choose an equation or situation:
The zero level for gravitational potential energy is chosen to be at the surface of the water. The total potential energy is the sum of the gravitational and elastic potential energy.

- \( PE_{\text{tot}} = PE_g + PE_{\text{elastic}} \)
- \( PE_g = mgh \)
- \( PE_{\text{elastic}} = \frac{1}{2} kx^2 \)

**SOLVE**

Substitute the values into the equations and solve:

- \( PE_g = (70.0 \text{ kg})(9.81 \text{ m/s}^2)(6.0 \text{ m}) = 4.1 \times 10^3 \text{ J} \)
- \( PE_{\text{elastic}} = \frac{1}{2} (71.8 \text{ N/m})(29.0 \text{ m})^2 = 3.02 \times 10^4 \text{ J} \)
- \( PE_{\text{tot}} = 4.1 \times 10^3 \text{ J} + 3.02 \times 10^4 \text{ J} \)
- \( PE_{\text{tot}} = 3.43 \times 10^4 \text{ J} \)

**CHECK YOUR WORK**

One way to evaluate the answer is to make an order-of-magnitude estimate. The gravitational potential energy is on the order of \( 1 \times 10^2 \text{ kg} \times 10 \text{ m/s}^2 \times 10 \text{ m} = 10^4 \text{ J} \). The elastic potential energy is on the order of \( 1 \times 10^2 \text{ N/m} \times 10^2 \text{ m}^2 = 10^4 \text{ J} \). Thus, the total potential energy should be on the order of \( 2 \times 10^4 \text{ J} \). This number is close to the actual answer.

**Problem Solving Tips and Tricks**

Choose the zero potential energy location that makes the problem easiest to solve.

**DECONSTRUCTING PROBLEMS**

The two numbers in scientific notation that are added get \( PE_{\text{tot}} \) cannot be added directly, since the exponents are different. To add them, we must first increase the power of the exponent in the first term to 4. To make this change, follow the step below:

- \( PE_g = 4.1 \times 10^3 = (0.41 \times 10) \times 10^4 = 0.41 \times 10^4 \)

Now, we can combine both terms:

- \( PE_{\text{tot}} = 0.41 \times 10^4 + 3.02 \times 10^4 = (0.41 + 3.02)10^4 = 3.43 \times 10^4 \)

---

**Answers**

- a. 9.81 J
- b. 2.00 J
- c. 11.81 J

**PROBLEM GUIDE D**

Use this guide to assign problems.

- **SE** = Student Edition Textbook
- **PW** = Sample Problem Set I (online)
- **PB** = Sample Problem Set II (online)

**Solving for:**

- **PE**
  - **SE** Sample, 1–3; Ch. Rvw. 23–25, 37
  - **PW** 7–9
  - **PB** 9–10
- **k**
  - **PW** 10
  - **PB** Sample, 1–3
- **h or d**
  - **PW** 4–6, 10
  - **PB** 4–6
- **m**
  - **PW** Sample, 1–3
  - **PB** 7–8

*Challenging Problem*
Reviewing Main Ideas

1. A pinball bangs against a bumper, giving the ball a speed of 42 cm/s. If the ball has a mass of 50.0 g, what is the ball's kinetic energy in joules?

2. A student slides a 0.75 kg textbook across a table, and it comes to rest after traveling 1.2 m. Given that the coefficient of kinetic friction between the book and the table is 0.34, use the work–kinetic energy theorem to find the book's initial speed.

3. A spoon is raised 21.0 cm above a table. If the spoon and its contents have a mass of 30.0 g, what is the gravitational potential energy associated with the spoon at that height relative to the surface of the table?

Critical Thinking

4. What forms of energy are involved in the following situations?
   a. A bicycle coasting along a level road
   b. Heating water
   c. Throwing a football
   d. Winding the mainspring of a clock

5. How do the forms of energy in item 4 differ from one another? Be sure to discuss mechanical versus nonmechanical energy, kinetic versus potential energy, and gravitational versus elastic potential energy.

Answers to Section Assessment

1. 4.4 \times 10^{-3} \text{ J}
2. 2.8 \text{ m/s}
3. 6.18 \times 10^{-2} \text{ J}
4. a. kinetic energy
   b. nonmechanical energy
   c. kinetic energy, gravitational potential energy
   d. elastic potential energy
5. The heated water is an instance of nonmechanical energy, because its mass is not displaced with a velocity or with respect to a zero position, as would be the case for the various types of mechanical energy. The bicycle and football both have masses in motion, so they have kinetic energy. The wound spring has been displaced from its relaxed position and so has elastic potential energy, while the football is above the ground and therefore has a gravitational potential energy associated with it.