Frames of Reference

If you are moving at 80 km/h north and a car passes you going 90 km/h north, to you the faster car seems to be moving north at 10 km/h. Someone standing on the side of the road would measure the velocity of the faster car as 90 km/h north. This simple example demonstrates that velocity measurements depend on the frame of reference of the observer.

Velocity measurements differ in different frames of reference.

Observers using different frames of reference may measure different displacements or velocities for an object in motion. That is, two observers moving with respect to each other would generally not agree on some features of the motion.

Consider a stunt dummy that is dropped from an airplane flying horizontally over Earth with a constant velocity. As shown in Figure 4.1(a), a passenger on the airplane would describe the motion of the dummy as a straight line toward Earth. An observer on the ground would view the trajectory of the dummy as that of a projectile, as shown in Figure 4.1(b). Relative to the ground, the dummy would have a vertical component of velocity (resulting from free-fall acceleration and equal to the velocity measured by the observer in the airplane) and a horizontal component of velocity given to it by the airplane's motion. If the airplane continued to move horizontally with the same velocity, the dummy would enter the swimming pool directly beneath the airplane (assuming negligible air resistance).

**FIGURE 4.1**

**Frames of Reference** When viewed from the plane (a), the stunt dummy (represented by the maroon dot) falls straight down. When viewed from a stationary position on the ground (b), the stunt dummy follows a parabolic projectile path.

**DIFFERENTIATED INSTRUCTION**

**INCLUSION**

Matching one moving object with another moving object can help visual learners understand relative velocity. Put a list of moving objects such as the ones to the right on the board. Ask students to name some moving objects that can be paired with one of the examples in order to establish a case for relative velocity. For example, students can match the motion of a moving boat with a person swimming in the water.

Examples:

1. moving boat
2. moving elevator
3. motion of a planet in the Solar System
4. walking passenger in a train
5. rolling of a ball bearing
Relative Velocity

The case of the faster car overtaking your car was easy to solve with a minimum of thought and effort, but you will encounter many situations in which a more systematic method of solving such problems is beneficial. To develop this method, write down all the information that is given and that you want to know in the form of velocities with subscripts appended.

\[ v_{se} = +80 \text{ km/h north} \] (Here the subscript \( se \) means the velocity of the slower car with respect to Earth.)

\[ v_{fe} = +90 \text{ km/h north} \] (The subscript \( fe \) means the velocity of the fast car with respect to Earth.)

We want to know \( v_{fs} \), which is the velocity of the fast car with respect to the slower car. To find this, we write an equation for \( v_{fs} \) in terms of the other velocities, so on the right side of the equation the subscripts start with \( f \) and eventually end with \( s \). Also, each velocity subscript starts with the letter that ended the preceding velocity subscript.

\[ v_{fs} = v_{fe} + v_{es} \]

The boldface notation indicates that velocity is a vector quantity. This approach to adding and monitoring subscripts is similar to vector addition, in which vector arrows are placed head to tail to find a resultant.

We know that \( v_{es} = -v_{se} \) because an observer in the slow car perceives Earth as moving south at a velocity of 80 km/h while a stationary observer on the ground (Earth) views the car as moving north at a velocity of 80 km/h. Thus, this problem can be solved as follows:

\[ v_{fs} = v_{fe} + v_{es} = v_{fe} - v_{se} \]

\[ v_{fs} = (+90 \text{ km/h north}) - (+80 \text{ km/h north}) = +10 \text{ km/h north} \]

When solving relative velocity problems, follow the above technique for writing subscripts. The particular subscripts will vary depending on the problem, but the method for ordering the subscripts does not change. A general form of the relative velocity equation is \( v_{fe} = v_{ab} + v_{bc} \). This general form may help you remember the technique for writing subscripts.

Answers

Conceptual Challenge

1. Greater than, because the elevator is accelerating upward toward the ball as it falls
2. The plane's velocity is slower relative to the moving carrier when it approaches from the stern.

Teaching Tip

Relative velocity can also be shown as the difference of two vectors.

\[ v_{fs} = v_{fe} - v_{se} \]

Another way of stating this equation is that the relative velocity of one moving object to another is the difference between their velocities relative to some common reference point.

You may want to demonstrate on the board that this equation works for noncollinear velocities, as in Sample Problem F on the next page.

Conceptual Challenge

1. Elevator Acceleration  A boy bounces a rubber ball in an elevator that is going down. If the boy drops the ball as the elevator is slowing down, is the magnitude of the ball's acceleration relative to the elevator less than or greater than the magnitude of its acceleration relative to the ground?
2. Aircraft Carrier  Is the velocity of a plane relative to an aircraft carrier slower when it approaches from the stern (rear) or from the bow (front)?
Relative Velocity

Sample Problem F  A boat heading north crosses a wide river with a velocity of 10.00 km/h relative to the water. The river has a uniform velocity of 5.00 km/h due east. Determine the boat’s velocity with respect to an observer on shore.

1 ANALYZE

Given:

\( v_{bw} = 10.00 \text{ km/h due north} \)  
(velocity of the boat, \( b \), with respect to the water, \( w \))

\( v_{we} = 5.00 \text{ km/h due east} \)  
(velocity of the water, \( w \), with respect to Earth, \( e \))

Unknown:

\( v_{be} = ? \)  
\( \theta = ? \)

Diagram:  
See the diagram on the right.

2 PLAN

Choose an equation or situation:

To find \( v_{be} \), write the equation so that the subscripts on the right start with \( b \) and end with \( e \).

\[ v_{be} = v_{bw} + v_{we} \]

We use the Pythagorean theorem to calculate the magnitude of the resultant velocity and the tangent function to find the direction.

\[ (v_{be})^2 = (v_{bw})^2 + (v_{we})^2 \]

\[ \tan \theta = \frac{v_{we}}{v_{bw}} \]

Rearrange the equations to isolate the unknowns:

\[ v_{be} = \sqrt{(v_{bw})^2 + (v_{we})^2} \]

\[ \theta = \tan^{-1} \left( \frac{v_{we}}{v_{bw}} \right) \]

3 SOLVE

Substitute the known values into the equations and solve:

\[ v_{be} = \sqrt{(10.00 \text{ km/h})^2 + (5.00 \text{ km/h})^2} \]

\[ v_{be} = 11.18 \text{ km/h} \]

\[ \theta = \tan^{-1} \left( \frac{5.00 \text{ km/h}}{10.0 \text{ km/h}} \right) \]

\[ \theta = 26.6^\circ \]

4 CHECK YOUR WORK  
The boat travels at a speed of 11.18 km/h in the direction 26.6° east of north with respect to Earth.

Problem Solving

TAKE IT FURTHER

Tell students that the rules for determining relative velocity break down as one approaches speeds close to the speed of light. The speed of light, \( 3.00 \times 10^8 \text{ m/s} \), is constant in a vacuum. If an object, such as a spaceship, approached Earth at 0.5 the speed of light, the lights from the headlights of the spaceship would not approach Earth at 1.5 times the speed of light. Instead, they would travel at the speed of light.
Relative Velocity (continued)

Practice

1. A passenger at the rear of a train traveling at 15 m/s relative to Earth throws a baseball with a speed of 15 m/s in the direction opposite the motion of the train. What is the velocity of the baseball relative to Earth as it leaves the thrower’s hand?

2. A spy runs from the front to the back of an aircraft carrier at a velocity of 3.5 m/s. If the aircraft carrier is moving forward at 18.0 m/s, how fast does the spy appear to be running when viewed by an observer on a nearby stationary submarine?

3. A ferry is crossing a river. If the ferry is headed due north with a speed of 2.5 m/s relative to the water and the river’s velocity is 3.0 m/s to the east, what will the boat’s velocity relative to Earth be? (Hint: Remember to include the direction in describing the velocity.)

4. A pet-store supply truck moves at 25.0 m/s north along a highway. Inside, a dog moves at 1.75 m/s at an angle of 35.0° east of north. What is the velocity of the dog relative to the road?

Reviewing Main Ideas

1. A woman on a 10-speed bicycle travels at 9 m/s relative to the ground as she passes a little boy on a tricycle going in the opposite direction. If the boy is traveling at 1 m/s relative to the ground, how fast does the boy appear to be moving relative to the woman?

2. A girl at an airport rolls a ball north on a moving walkway that moves east. If the ball’s speed with respect to the walkway is 0.15 m/s and the walkway moves at a speed of 1.50 m/s, what is the velocity of the ball relative to the ground?

Critical Thinking

3. Describe the motion of the following objects if they are observed from the stated frames of reference:
   a. a person standing on a platform viewed from a train traveling north
   b. a train traveling north viewed by a person standing on a platform
   c. a ball dropped by a boy walking at a speed of 1 m/s viewed by the boy
   d. a ball dropped by a boy walking 1 m/s as seen by a nearby viewer who is stationary

SECTION 4 FORMATIVE ASSESSMENT

Answer to Section Assessment

1. 10 m/s (in the opposite direction)
2. 1.51 m/s at 5.7° north of east
3. a. south with a speed equal to the train’s speed
   b. moves north
   c. appears to fall straight down
   d. moves in a parabola

PROBLEM GUIDE F

Use this guide to assign problems.
SE = Student Edition Textbook
PW = Sample Problem Set I (online)
PB = Sample Problem Set II (online)

Solving for:

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*Challenging Problem

Answers

Practice F

1. 0 m/s
2. 14.5 m/s (in the direction that the aircraft carrier is moving)
3. 3.90 m/s at (4.0 × 10°)° north of east
4. 26.4 m/s at 2.17° east of north

Assess and Reteach ▼

Assess  Use the Formative Assessment on this page to evaluate student mastery of the section.

Reteach  For students who need additional instruction, download the Section Study Guide.

Response to Intervention  To reassess students’ mastery, use the Section Quiz, available to print or to take directly online at HMDScience.com.